Platform of Platforms in Ride-Hailing

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Abstract

The platform economy has reshaped many business models. Platforms serve various purposes, including communication, networking, gaming, and services. However, many essential activities such as order management and payment are common to them, creating a new space for a platform of platforms (PoP). Despite the importance and fast development of PoP, many important research questions still need to be answered: how a PoP affects platform entry, and how different ownership structures affect the market power of platforms. This paper fills in the gap by analyzing the impacts of PoPs theoretically and empirically. We construct the first multi-sided market model of PoP, study its properties, and derive model implications. Second, we evaluate how PoP affects market equilibrium with an estimated structural model and analyze welfare implications for workers and consumers. Our empirical analysis studies PoP in the Chinese ride-hailing industry. Using data on prices and service availability for all ride-hailing platforms in all Chinese cities, we document three benefits of having a PoP. First, PoP has its own customer base, bringing extra network effects for all its affiliated operating platforms. Second, PoP pools all the customers and drivers together. As a result, once an operating platform joins the PoP, it can be matched with a larger potential set of customers and enjoy a greater cross-side network effect. Third, PoP offers the operating platforms an opportunity to enter the market at a lower cost. If an operating platform has a high entry cost, it will be more cost-effective to enter the market by joining the PoP rather than developing its own app.

Keywords: Two-Sided Platform, Market Power, Platform Incentive Scheme, Vertical Structure, Fairness in Algorithms

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1 Introduction

The platform economy has reshaped many business models. In addition to consumer activities and sales models, the platform economy also impacts the nature of jobs and the workforce. Different platforms serve various purposes, including communication, networking, gaming, and services. However, many essential activities such as order management and payment are common to them, creating a new space for a platform of platforms (PoP). A platform of platforms provides internet-level commerce services to support such common activities on other platforms. Amazon's AWS is one example of existing PoPs. The demand for PoPs is fast growing in many areas. For instance, Stripe is a platform that plugs into other platforms and provides payment services. Okta allows companies to offer simple and secure sign-on.

Moreover, the discussion surrounding platform of platforms becomes increasingly relevant due to recent government policies. Numerous countries have taken steps to strengthen their standards in safeguarding data and privacy. A prominent example is the implementation of the General Data Protection Regulation (GDPR) by the European Union (EU) in 2018. The protection regulations have three significant impacts on platforms: they raise operational costs for platforms, restrict the growth of smaller platforms by prohibiting data sharing, and steer consumer and advertiser preferences towards larger platforms. According to studies conducted by Geradin, Katsifis and Karanikioti (2020) and Li, Yu and He (2019), the increased operational costs imposed on tech firms within the EU due to GDPR can potentially lead to their withdrawal from the sector, resulting in reduced competition. Furthermore, the GDPR imposes limitations on the growth of small platforms (Jia, Jin and Wagman, 2018). Additionally, the preference of customers and advertisers tends to shift towards larger platforms after the introduction of GDPR. Sharma, Sun and Wagman (2019) demonstrate that most small platforms experience a decline in profits once privacy laws are established. Given the impacts of such privacy laws and how they favor large platforms over small ones, there has been a growing call for companies to collaborate and establish a "platform-of-platforms". This PoP concept holds the potential to offer a comprehensive and on-demand platform with reduced costs.

Despite the importance and fast development of PoP, many important research questions still need to be answered: for example, how PoPs affect platform entry and how different ownership structures affect the market power of platforms and PoPs. There have been discussions on multi-sided markets (Hagiu and Wright, 2015; Rochet and Tirole, 2006; Weyl, 2010), and how ownership structures such as vertical integration affect market outcomes (Crawford et al., 2018; Hagiu et al., 2022). However, none of the analyses is directly applicable to

PoPs. Therefore, we aim to fill in the gap by analyzing the impacts of PoPs theoretically and empirically. We make the following contributions: first, we aim to construct the first structural model of PoP, study its properties, and derive model implications. Second, with an estimated structural model, we evaluate how PoP affects market equilibrium and analyze welfare implications on drivers and consumers. Last, we study the design and regulation of PoP.

Our empirical study focuses on the ride-hailing industry in China, specifically examining the context of a prominent platform-of-platforms (PoP) called AutoNavi. Initially established as a map navigation application, AutoNavi transitioned into a PoP in 2017 by hosting multiple operating platforms and offering ride-hailing services. AutoNavi serves as an intermediary between consumers and drivers affiliated with operating platforms. Passengers utilize AutoNavi to submit ride requests, providing their pickup and drop-off locations. Concurrently, operating platforms collaborate with AutoNavi, connecting their drivers to AutoNavi's platform. This integration enables AutoNavi to match passenger orders with drivers from specific operating platforms, facilitating the completion of ride requests. In this study, we aim to empirically investigate the impact of a PoP within the ride-hailing industry.

A PoP in the ride-hailing industry has several unique features. First, PoP has its own customer base, bringing extra network effects for all its affiliated operating platforms. Second, PoP pools all the customers and drivers together. As a result, once an operating platform joins the PoP, it can be matched with a larger potential set of customers and enjoy a greater cross-side network effect. Third, PoP offers the operating platforms an opportunity to enter the market at a potentially lower cost. If an operating platform has a high entry cost, it will be more cost-effective to enter the market by joining the PoP rather than developing its own app. Our structural model aims to include all three features.

To understand the impact of PoP, We build a two-sided market model for the ride-hailing industry. At the bottom level, drivers make decisions regarding which operating platform to work for, taking into consideration the wage rates offered by each platform. At the intermediate level, each operating platform determines its wage rates and ride fares in order to compete for both drivers and customers. Furthermore, potential entrants assess whether to enter the market through PoP or not. PoP itself determines its commission rate per usage. At the top level, customers choose which operating platform to use based on the availability of drivers and ride fares. Our model highlights the critical role of PoP, firm heterogeneity, and elastic demand.

Using the estimated model, our objective is to perform counterfactual experiments to explore the following questions: Firstly, we examine the influence on market equilibrium when a PoP is introduced, comparing it to a scenario without a PoP. We investigate the welfare implications for both drivers and consumers once a PoP is established. Additionally, we analyze the effects of PoP on entry and exit decisions for operating platforms, assessing whether the presence of a PoP leads to increased or decreased market concentration. Next, we will alter the incentive scheme of the PoP. If the PoP is socially benevolent compared to profit-maximizing, how do the answers to the previous question change? As suggested in the discussions related to the General Data Protection Regulations in Europe, some people argue that the government should initialize specific super platforms to facilitate communication and better utilize different operating platforms. Anecdotal evidence shows that if the PoP owns shares in the operating platform, the algorithm of the PoP may prioritize the operating platform in trip assignments. To understand the welfare implication of a fair algorithm, we will simulate a new market equilibrium in which PoP treats all companies equally and levels the playing field for the firms. In general, it elaborates on how "fair" algorithms change market equilibrium. Lastly, in our benchmark model, each operating platform independently determines its ride fares. However, we introduce the concept of "centralized" pricing by the PoP and simulate its potential effects on the market equilibrium. Through this simulation, we aim to assess the implications of centralized pricing on the overall market equilibrium.

2 Industry Background

Over the past ten years, the ride-hailing industry in China has undergone remarkable expansion and advancement, fundamentally reshaping the transportation landscape in the country. The Ministry of Transport's data reveals that by the end of 2021, the annual number of completed ride-hailing orders had surged to an impressive 8.32 billion. Moreover, a total of 258 online operating platforms were granted licenses across the country to provide ride-hailing services, with the number of drivers exceeding 3.9 million individuals.[1] Platforms also grow alongside the ride-hailing industry. Operating platforms, such as Didi, Gaode, Caocao, T3, and others, connect riders and drivers. Recently, some platforms have adopted a new business model where they connect riders with operating platforms and rely on these operating platforms to manage drivers. We refer to these platforms as Platform of Platforms (PoP).

In our empirical analysis, we focus on examining a prominent example of a PoP called AutoNavi, which is widely recognized as Gaode Map. Initially launched as a navigation and map application that offered real-time traffic updates and public transportation guidance, AutoNavi has evolved into the dominant player in the Chinese map navigation market, similar to Google Maps' position in the United States. In 2017, AutoNavi began operating as a PoP by integrating third-party ride-hailing operating platforms into its ecosystem. The pri-

[1] https://new.qq.com/rain/a/20220228A07JV600.

mary objective was to establish partnerships with external companies, leveraging AutoNavi's location-based services to enhance the user experience and generate revenue. Unlike Uber or Lyft, AutoNavi itself does not provide direct ride-hailing services. Instead, customers can use AutoNavi to request rides from the ride-hailing platforms hosted by AutoNavi.

Figure 1 illustrates the market structure, depicting the process from the customer's perspective. Initially, customers have the option to choose either the PoP or one of the existing operating platforms. If the customer uses the existing operating platform's application directly, the operating platform will match customers with drivers who are affiliated with that specific operating platform. In this case, the operating platform handles the process of matching the customer with a driver from its own network of affiliated drivers. In contrast, if they choose the PoP, they will receive information about ride fares and service availability from each operating platform affiliated with the PoP. Afterwards, customers will be matched with one of the operating platforms hosted by the PoP. The operating platform is then responsible for determining the ride fare and assigning a driver for the pick-up. Furthermore, the operating platform has control over dispatching and routing decisions for each ride.

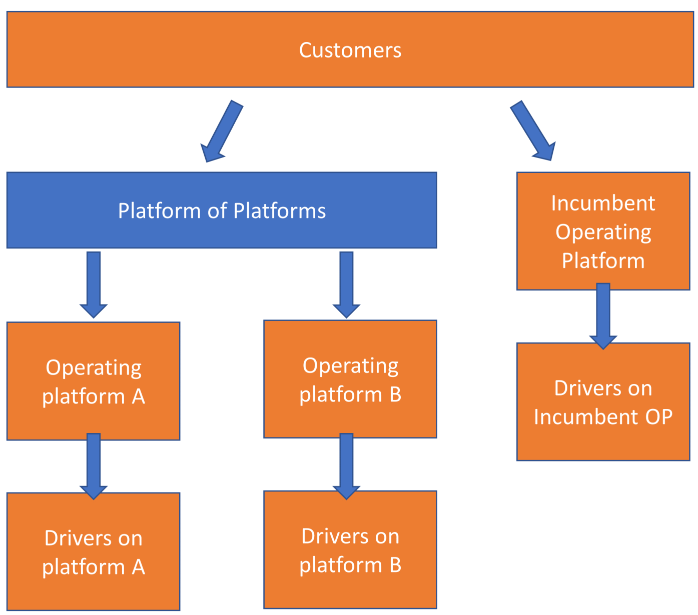


Figure 1: Illustration of the Market Structure

In this context, we will provide a more detailed explanation of how the PoP operates. To begin with, the customer enters the departure and arrival addresses within AutoNavi. Subsequently, AutoNavi forwards these addresses to all affiliated third-party operating platforms. Each operating platform associated with the PoP then computes and transmits the fare and service availability information back to the AutoNavi application. AutoNavi displays the price and service availability offered by each platform to the customer. From the available options, customers can make their selection, and AutoNavi proceeds to transmit the ride request to the chosen operating platforms. The third-party operating platforms initiate the search for available drivers and provide driver information, including location and driver quality, to the PoP. The PoP then assigns the customer to one of the drivers and the associated operating platform. This selected operating platform will be responsible for providing the ride service to the customer. It is important to note that all the aforementioned information transfers occur within a very short timeframe, ensuring a swift and efficient process.

Figure 2 provides a comparison of the interfaces between AutoNavi and Uber. In both applications, customers begin by entering their departure and arrival addresses. However, there are differences in the subsequent steps. For Uber customers (shown in the right figure), they have the option to select from various types of services with different fare structures, such as UberX, UberXL, Comfort, and more. Once the service type is chosen, the order is then distributed to nearby Uber drivers. In contrast, AutoNavi (shown in the left figure) displays a set of operating platforms along with estimated fares for each platform. Examples of these operating platforms include DiDi, YangGuang, and AA Chuxing as depicted in the figure. AutoNavi riders can make their selection by clicking the checkbox next to the estimated fare for each operating platform. This allows them to choose the set of operating platforms they prefer to ride with. Once the selection is made, the order is distributed to nearby drivers associated with the selected operating platforms.

Although AutoNavi is a leading ride-hailing provider in China, it does not have drivers affiliated with its platform, and it does not have a dedicated app specifically designed for its drivers. As a result, the operating platforms associated with AutoNavi handle the process of receiving ride orders and providing services through their own driver-side apps. The operative platforms establish a connection with AutoNavi's ride-hailing service through negotiation and cooperation with AutoNavi. As AutoNavi has the authority to allocate ride orders, the relationship with AutoNavi can have an impact on the quantity and quality of orders allocated to a particular platform. It is important to note that the orders received by different platforms may vary significantly. While larger platforms may have their own passenger-side applications, the orders allocated by AutoNavi serve as a primary source for smaller platforms to acquire new users and expand their customer base.

The PoP model provides a significant advantage for small operating platforms in competing with larger platforms. First, the PoP model indeed provides infrastructure that facilitates the entry of small operating platforms into the ride-hailing market. AutoNavi, through its subsidiary corporation Bailongma, offers a Software-as-a-Service (SAAS) solution specifi-

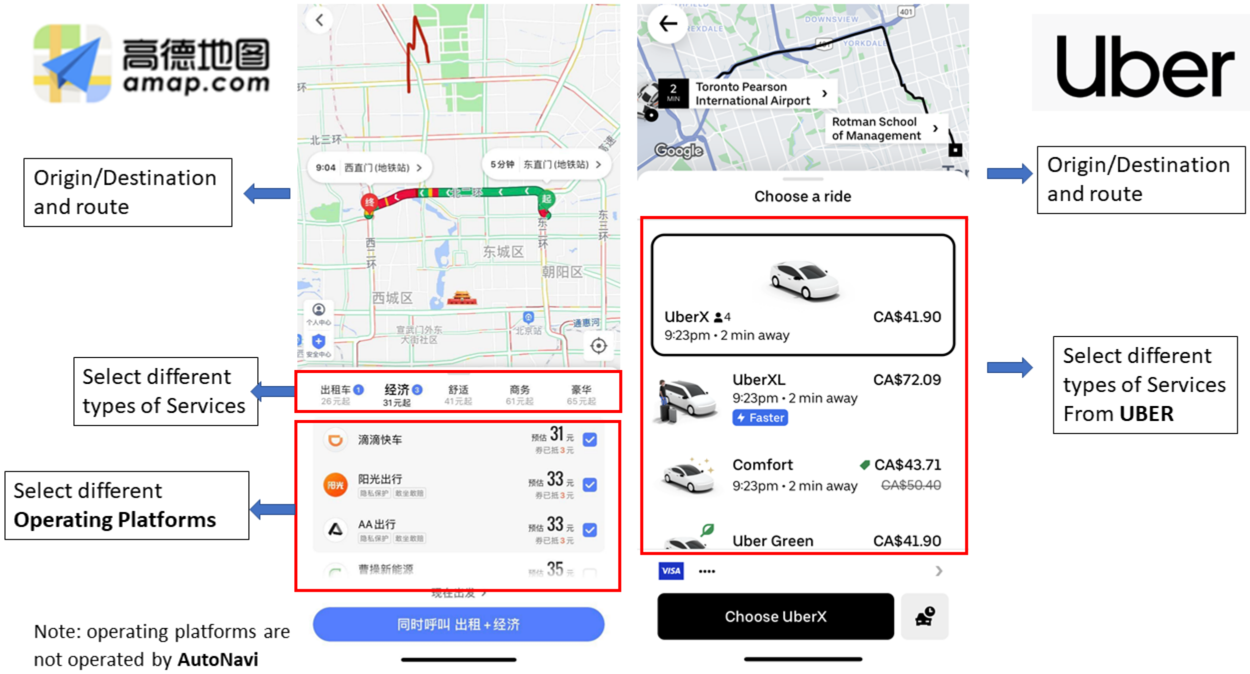


Figure 2: Compare AutoNavi and Uber interfaces

cally designed for operating platforms. Bailongma assists operating platforms by designing apps, supporting operational processes, providing financial services, and managing customer service. This comprehensive support system helps streamline the operations of the operating platforms, enabling them to focus on providing efficient ride-hailing services. Furthermore, in terms of government relationships, AutoNavi collaborates with other companies to assist in obtaining the necessary ride-hailing company operating licenses in each local city.[2] This partnership helps navigate the complex regulatory landscape and ensures compliance with local regulations, which is crucial for the smooth operation of the participating operating platforms.[3] Second, small operating platforms can benefit from the network effect created by the PoP. In the ride-hailing market, which is characterized by its two-sided nature, smaller platforms often face challenges due to having fewer drivers and being less appealing to customers. However, the PoP model helps address these challenges by aggregating customer demand. With AutoNavi's extensive user base of over one hundred million daily active users, the PoP can consolidate and channel demand from these customers to the affiliated operating platforms. This aggregation of demand provides smaller platforms with access to a larger

[2]For instance, Xiao Ka Ke Ji https://www.rvakva.com/.

[3]It's worth noting that there is an approximate annual fee of 600,000 RMB for an operating platform to join AutoNavi's PoP. This fee covers the benefits and services provided by AutoNavi, including access to its infrastructure, user base, and support systems.

pool of potential customers, leveling the playing field and enabling them to compete more effectively with larger platforms. By leveraging the vast user network of the PoP, smaller operating platforms can increase their visibility, attract more customers, and expand their market presence.

Lastly, operating as a PoP instead of an operating platform offers several advantages to the platform itself. Firstly, it is relatively easy for AutoNavi to expand by adding operating platforms, similar to expanding through franchisees. It doesn't have to bear the substantial costs and risks associated with entering new markets. Second, by allowing operating platforms to join the PoP, AutoNavi can also capitalize on the network effect, leveraging the existing user base and infrastructure of the operating platforms. Third, as a PoP, AutoNavi doesn't need to directly engage with drivers. This helps mitigate certain risks associated with regulations and accidents that may arise in the ride-hailing industry. By maintaining a level of separation between AutoNavi and the drivers, any negative incidents or issues that may occur are less likely to directly impact AutoNavi's platform reputation. This can help safeguard AutoNavi's brand image and reputation in the market.

Overall, operating as a PoP offers advantages in terms of cost-effective expansion, network effect and risk management, supporting AutoNavi's fast growth in the ride-hailing industry. As of June 2022, AutoNavi has established partnerships with 221 operating platforms across China. This extensive network allows AutoNavi's ride-hailing service to be accessible in over 200 cities throughout the country. The AutoNavi Map App has garnered a substantial user base, with a daily active user count of 120 million individuals. Out of these users, 13 million rely on AutoNavi to request ride-hailing services. AutoNavi has emerged as a significant competitor in the ride-hailing market, positioning itself alongside the leading incumbent, DiDi Global.

3 Data

Our analysis relies on two primary datasets. Firstly, we have gathered data on prices and service availability across all Chinese cities for every ride-hailing platform. For each specific origin–destination trip, we obtain the price charged by each ride-hailing platform and the corresponding service availability. Secondly, we have access to all completed transactions that took place in December 2018 within a major city in China. For each transaction, we observe details such as the departure and destination points, distance traveled, duration of passenger pickup and transportation, and the price paid by the rider. With our comprehensive transaction-level data, we can observe fluctuations in order quantities across different times of the day and various geographic areas within the city.

Table 1 shows the general market structure in the ride-hailing industry in China. On average, there are 14 operating platforms in each city, with a median of 12. Operating platforms vary greatly from one another. The largest operating platform serves almost all cities, while 94 out of the 275 operating platforms only serve one city.

Table 1: Summary Statistics of Operating Platforms

	Mean	Std. Dev.	25 Pctl	Median	75 Pctl
# Operating Platforms in Each City	14	8	9	12	16
# Cities Each Operating Platform Serves	18	41	1	3	13

Table 2 provides an overview of the transaction data. The unit of observation is measured at the driver-hour level. On average, drivers serve 1.9 orders per hour and earn a total of 50 CCY. The number of orders ranges from 1 to 3 between the 25th and 75th percentiles. Typically, drivers generally only spend half the time transporting riders. On average, drivers spend approximately 10 minutes for passenger pickups and an additional 19 minutes waiting for new orders.

Table 2: Summary Statistics (Driver-Hour)

	Mean	Std. Dev.	Min	25 Pctl	Median	75 Pctl	Max
Hourly Wage (CCY)	49.98	24.52	0	32.83	47.42	62.74	286.86
Earning Time (minutes)	30.60	12.01	0	21	31	40	60
Pickup time (minutes)	10.62	6.67	0	6	10	15	60
Idle Time (minutes)	18.78	14.32	0	6	17	29	60
Number of Orders	1.89	1.11	0	1	2	3	9
Distance (km)	14.11	7.41	0	8.78	13.1	18.2	94.13
Number of Observations				4,182,318			

Because our transaction data is obtained from a single major city in China, in order to accurately estimate the average prices and service availability of every ride-hailing platform in all other cities, we rely on trip simulations that leverage the observed characteristics of each geographical area. Specifically, we employ a gravity model to model the number of trips between different geographical areas within a given city. The model can be represented as:

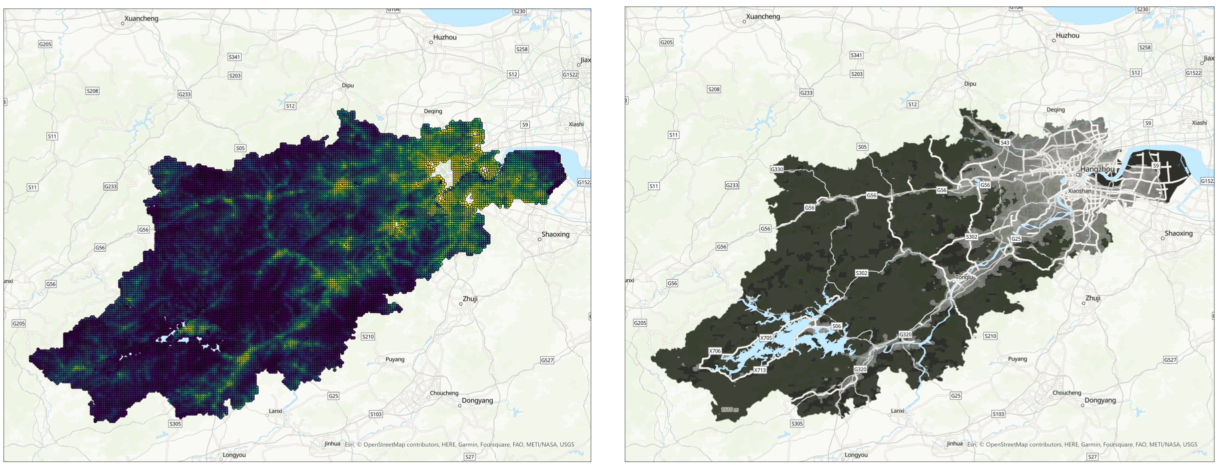
$$Y_{ij} = GX_i^{\beta_1} X_j^{\beta_2} dist_{ij}^{\beta_3} \epsilon_{ij} \tag{1}$$

In the equation, Y_{ij} denotes the number of trips between each location i and j, X_i and X_j represent the characteristics of locations i and j, respectively, and ϵ_{ij} is an error term assumed to follow a log-normal distribution, which is unknown to econometricians.

By applying the natural logarithm transformation, equation 1 can be estimated in a log-linear form as follows:

$$\log Y_{ij} = \log G + \beta_1 \log X_i + \beta_2 \log X_j + \beta_3 \log dist_{ij} + \log \epsilon_{ij} \tag{2}$$

In our numerical analysis, we focus on two geographic factors to assess each location's level of economic activity: PoPulation density and nighttime light. Figure 3 illustrates the observed PoPulation density and nighttime light for the city where we possess transaction information. Utilizing this data, we can proceed with estimating the parameters in Equation 2 based on the observed transaction data.



(a) PoPulation Density (b) Nighttime Light

Figure 3: Characteristics Employed in Trip Simulation

To facilitate the simulation process, we divide each city into $1km \times 1km$ grids. By employing the estimated parameters from Equation 1, we can simulate the number of trips for each pair of grids. Additionally, since we possess price information and service availability data for each grid-pair, we can combine this information with the simulated trip numbers. This enables us to compute the average price and service availability for any ride-hailing platform in any city across China.

4 Model

Our model consists of four agents, namely consumers, drivers, operating platforms, and a platform of platform (PoP). Consumers select an operating platform to fulfill their ride requests. Drivers, who are single-homing, decide which operating platform to join. In addition to an incumbent operating platform I, potential entrants can also enter the market through the PoP. The operating platforms determine the ride fare, and the PoP aims to maximize the commission fee earned from all the operating platforms that are associated with it.

4.1 Set-up

Demand

Each ride i chooses whether to use the ride-hailing service or not, based on the utility function

$$U_{ik} = \alpha P_k + \beta \frac{M_k}{N_k} + \xi_k + \mathbb{1}[k \in G]\xi_G + \varepsilon_{ik} \tag{3}$$

where P_k is the price charged by operating platform k, M_k and N_k represents platform k's number of drivers and riders respectively, ξ_k is the platform fixed effect for consumers, ξ_G is the fixed effect of the PoP, and $\mathbb{1}[k \in G]$ is the indicator function of whether operating platform k is in the PoP or not. The interpretation of the utility function is that consumers care about the ride fare P_k, the availability of drivers $R_k = M_k/N_k$, and the fixed effects of each platform k.

For the incumbent, the total demand is then derived as

$$N_I(\boldsymbol{P}) = N \cdot \frac{\exp\left(\alpha P_I + \beta \frac{M_I}{N_I} + \xi_I\right)}{1 + \exp\left(\alpha P_I + \beta \frac{M_I}{N_I} + \xi_I\right) + \sum_k \exp\left(\alpha P_k + \beta \frac{M_k}{N_k} + \xi_k + \xi_G\right)}$$

where $\boldsymbol{P}$ is the vector of prices for all the operating platforms. k is the index indicating each entrant.

For each entrant e, the total demand is derived as

$$N_e(\boldsymbol{P}) = N \cdot \frac{\exp\left(\alpha P_e + \beta \frac{M_e}{N_e} + \xi_e + \xi_G\right)}{1 + \exp\left(\alpha P_I + \beta \frac{M_I}{N_I} + \xi_I\right) + \sum_k \exp\left(\alpha P_k + \beta \frac{M_k}{N_k} + \xi_k + \xi_G\right)}$$

Labor Supply

Each driver j chooses whether to work for platform k or not. The utility of driver j working for platform k is defined as

$$U_{jk} = \gamma(1-\tau)\frac{N_k}{M_k} P_k + \eta_k + \epsilon_{jk} \tag{4}$$

where γ is the normalization of the extreme-value type errors, τ is the fee charged by the operating platform, and η_k is the fixed effect of operating platform for drivers. The interpretation of the utility function for drivers is that drivers care about the total wage earned, which depends on the fees τ, the number of orders they will be assigned with (captured by N_k/M_k), and the ride fare P_k.

Therefore, the total number of drivers for incumbent platform I is derived as

$$M_I(\boldsymbol{P}) = M \cdot \frac{\exp\left(\gamma(1-\tau)\frac{N_I}{M_I}P_I + \eta_I\right)}{1 + \exp\left(\gamma(1-\tau)\frac{N_I}{M_I}P_I + \eta_I\right) + \sum_k \exp\left(\gamma(1-\tau)\frac{N_k}{M_k}P_k + \eta_k\right)}.$$

The total number of drivers for each entrant e is derived as

$$M_e(\boldsymbol{P}) = M \cdot \frac{\exp\left(\gamma(1-\tau)\frac{N_e}{M_e}P_e + \eta_e\right)}{1 + \exp\left(\gamma(1-\tau)\frac{N_I}{M_I}P_I + \eta_I\right) + \sum_k \exp\left(\gamma(1-\tau)\frac{N_k}{M_k}P_k + \eta_k\right)}.$$

Operating Platform's Decision

The objective of the incumbent platform I is:

$$\pi_I := \max_{P_I} \quad \tau P_I \cdot N_I(P_I, \boldsymbol{P}_{-I}) \tag{5}$$

where τ is the fee the operating platform charges to the drivers. We assume that τ is exogenously fixed at 20% per ride. N_I is the total demand of rides for platform I, P_I is the ride fare charged by the incumbent I, and $\boldsymbol{P}_{-I}$ is the vector of ride fares for all the other operating platforms in the market.

The objective of each entrant e if it enters the market is to maximize its profit:

$$\pi_e := \max_{P_e} \quad (\tau P_e - \rho) \cdot N_e(P_e, \boldsymbol{P}_{-e}) \tag{6}$$

ρ is the commission fee charged by the PoP. In the baseline model, we assume that $\rho = 0$. In the counterfactual analysis, we assume that the PoP maximizes its profit by optimizing over the commission fee ρ.

Remark 1. *Note the incumbent is a platform independent of the PoP, so it pays no fee to the PoP. Moreover, we restrict the platform's pricing decisions to rider fares. In general, a platform chooses both rider fares and its take rate. However, the focal market is moving towards a more transparent scheme with take rates simplified and posted. We will fix τ and ρ and focus on platforms' rider fare decisions in our analysis. Nevertheless, our framework* *is flexible to account for varying take rates.*

The platform of platform receives

$$\rho \sum_e N_e(P_e, P_{-e})$$

Entry

Regarding the entry decision, we assume that there is a group of fringe firms that can only enter through PoP. We assume that entry follows a Binomial-Poisson hierarchical model. More specifically, we assume that the number of potential entrant follows a Poisson distribution. Then each potential entrant endogenously choose whether to enter the market or not with probability p. Therefore, if we denote E = number of entrants and Y = number of potential entrants, we have

$$E|Y \sim binomial(Y, p), \quad Y \sim Poisson(\lambda)$$

It is easy to show that the unconditional distribution of E is also a Poisson distribution with parameter λp. Entrants are subject to the free entry condition with a fixed entry cost κ.

4.2 Theoretical Results

4.2.1 Equilibrium Existence

Theorem 1. *Given the prices $\boldsymbol{P}$ and τ, there exists an equilibrium $\{M_I(\boldsymbol{P}), M_e(\boldsymbol{P}), N_I(\boldsymbol{P}), N_e(\boldsymbol{P})\}$.*

We focus on the ride demand and labor supply model in this section. First, given the number of drivers and riders, the utility of riders is $U_{ik} = \alpha P_k + \beta \frac{M_k}{N_k} + \xi_k + \epsilon_{ik}$, and the utility of drivers is $U_{jk} = \gamma(1-\tau)\frac{N_k}{M_k} P_k + \eta_k + e_{jk}$, leading to firm-specific riders and drivers

$$N_k(\boldsymbol{P}) = N \cdot \frac{\exp\{\alpha P_k + \beta \frac{M_k}{N_k} + \xi_k\}}{1 + \sum_{k'} \exp\{\alpha P_{k'} + \beta \frac{M_{k'}}{N_{k'}} + \xi_{k'}\}}$$
$$M_k(\boldsymbol{P}) = M \cdot \frac{\exp\{\gamma(1-\tau)\frac{N_k}{M_k} P_k + \eta_k\}}{1 + \sum_{k'} \exp\{\gamma(1-\tau)\frac{N_{k'}}{M_{k'}} P_{k'} + \eta_{k'}\}}$$

where we expect $\alpha < 0$ representing downward-sloping ride demand, $\beta > 0$ representing appreciation of availability, and $\gamma > 0$ representing upward-sloping labor supply.

Given any $(\boldsymbol{M}, \boldsymbol{N})$, there is a unique $\boldsymbol{P}$ defined by

$$P_k = \frac{1}{\alpha}\Big[(\log N_k - \log N_0) - (\beta\frac{M_k}{N_k} + \xi_k)\Big]$$

Given any price vector $\boldsymbol{P}$, this is a continuous function of $[0, N]^K \times [0, M]^K$ to $[0, N]^K \times [0, M]^K$. Therefore, the Brouwer fixed point theorem guarantees at least one solution.

To study equilibrium uniqueness, we consider availability $R_k = M_k/N_k$.

$$R_k(\boldsymbol{P}) = R \cdot \frac{\exp\{\gamma(1-\tau)\frac{1}{R_k}P_k + \eta_k\}}{\exp\{\alpha P_k + \beta R_k + \xi_k\}} \cdot \frac{1 + \sum_{k'}\exp\{\alpha P_{k'} + \beta R_{k'} + \xi_{k'}\}}{1 + \sum_{k'}\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'} + \eta_{k'}\}} \tag{7}$$

which has a unique solution when $K = 1$ because the RHS strictly decreases in R_1, so it crosses the 45-degree line once and only once.

Now we consider $\beta > 0$ and $K > 1$, which is more interesting because it makes sense that riders prefer better availability and there is competition. We now transform the problem into a more tractable form. Fix the prices P and consider this system

$$\log R_k - \log R_0 = [\gamma(1-\tau)\frac{1}{R_k}P_k + \eta_k] - [\alpha P_k + \beta R_k + \xi_k] \tag{8}$$

$$f(R_0) = R\frac{1 + \sum_{k'}\exp\{\alpha P_{k'} + \beta R^*_{k'}(R_0) + \xi_{k'}\}}{1 + \sum_{k'}\exp\{\gamma(1-\tau)\frac{1}{R^*_{k'}(R_0)}P_{k'} + \eta_{k'}\}} \tag{9}$$

where $\alpha < 0, \beta > 0, \gamma > 0, \tau \in (0, 1)$, $P_k > 0$ and ξ_k, η_k are flexible. Note that(8)'s LHS is increasing and RHS is decreasing in R_k, so there is a unique solution given R_0. Denote it as $R^*_k(R_0)$. Notice that this solution is strictly increasing in R_0. Moreover, $f(\cdot)$ is strictly increasing in R_0. A fixed point of R_0, i.e., $f(R_0) = R_0$, represents a fixed point in the original problem and vice versa. So we can solve the model by solving the system (8) and (9), which is much simpler. Even if we cannot show equilibrium uniqueness, the counterfactual R_0 will likely be close to the original one if we make small changes in the counterfactual, guiding our numerical search. We can also exclude unreasonable ranges.

4.2.2 Uniqueness

Taking the derivate of (8) with respect to R_0 gives

$$\frac{R'_k}{R_k} - \frac{1}{R_0} = \gamma(1-\tau)\frac{-R'_k}{R_k^2}P_k - \beta R'_k \tag{10}$$

Solving for R_k' gives

$$R_k' = \frac{\frac{1}{R_0}}{\frac{1}{R_k} + \gamma(1-\tau)P_k\frac{1}{R_k^2} + \beta} = \frac{\exp\left[\left(\gamma(1-\tau)\frac{1}{R_k}P_k + \eta_k\right) - (\alpha P_k + \beta R_k + \xi_k)\right]}{1 + \gamma(1-\tau)P_k\frac{1}{R_k} + \beta R_k} > 0 \quad (11)$$

where we have ignored the star in $R_k^*(\cdot)$. The second equation follows from (8).

Lemma 1. *If R_0 goes to zero, $R_k = R_k^*(R_0)$ goes to zero and R_k' goes to infinity. If R_0 goes to infinity, R_k goes to infinity and R_k' goes to zero.*

Proof. If R_0 goes to zero, $R_k = R_k^*(R_0)$ goes to zero follows from (8) and R_k' goes to infinity follows from (11). Similarly, if R_0 goes to infinity, R_k goes to infinity follows from (8) and R_k' goes to zero follows from (11).

To see that "R_k' goes to infinity" when R_0 goes to zero, consider

$$\begin{aligned}
\lim_{R_k \downarrow 0} R_k'(R_0) &= \lim_{R_k \downarrow 0} \frac{\exp\left[\left(\gamma(1-\tau)\frac{1}{R_k}P_k + \eta_k\right) - (\alpha P_k + \beta R_k + \xi_k)\right]}{1 + \gamma(1-\tau)P_k\frac{1}{R_k} + \beta R_k} \\
&= \lim_{R_k \downarrow 0} \frac{\exp\left[\left(\gamma(1-\tau)\frac{1}{R_k}P_k + \eta_k\right) - (\alpha P_k + \xi_k)\right]}{1 + \gamma(1-\tau)P_k\frac{1}{R_k}} \\
&= \lim_{R_k \downarrow 0} \frac{\gamma(1-\tau)\frac{-1}{R_k^2}P_k \exp\left[\left(\gamma(1-\tau)\frac{1}{R_k}P_k + \eta_k\right) - (\alpha P_k + \xi_k)\right]}{\gamma(1-\tau)P_k\frac{-1}{R_k^2}} \\
&= +\infty
\end{aligned}$$

□

Now consider $f(R_0)$ and its first-order derivative

$$\begin{aligned}
f'(R_0) = {} & R\frac{\sum_{k'} \beta R_{k'}' \exp\{\alpha P_{k'} + \beta R_{k'} + \xi_{k'}\}}{1 + \sum_{k'} \exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'} + \eta_{k'}\}} \\
& + R\frac{1 + \sum_{k'} \exp\{\alpha P_{k'} + \beta R_{k'} + \xi_{k'}\}}{\left[1 + \sum_{k'} \exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'} + \eta_{k'}\}\right]^2}\left[\sum_{k'}\left(\gamma(1-\tau)\frac{R_{k'}'}{R_{k'}^2}P_{k'}\right)\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'} + \eta_{k'}\}\right] \\
> {} & 0
\end{aligned}$$

Lemma 2. *If R_0 (and R_k) goes to zero, $f(R_0)$ goes to zero and $f'(R_0)$ goes to infinity. If R_0 (and R_k) goes to infinity, $f(R_0)$ goes to infinity and $f'(R_0)$ goes to zero.*

Proof. It is easy to see that (9) converges to zero if R_0 (and R_k) goes to zero, and to infinity if R_0 (and R_k) goes to infinity.[4]

To see "$f'(R_0)$ goes to infinity" when R_k goes to zero, we consider its two terms separately. First, consider any small number $\epsilon > 0$

$$R\frac{\sum_{k'}\beta R'_{k'}\exp\{\alpha P_{k'}+\beta R_{k'}+\xi_{k'}\}}{1+\sum_{k'}\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'}+\eta_{k'}\}} = \frac{\sum_k \frac{\beta R}{1+\gamma(1-\tau)P_k\frac{1}{R_k}+\beta R_k}\exp\left[\left(\gamma(1-\tau)\frac{1}{R_k}P_k+\eta_k\right)\right]}{1+\sum_{k'}\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'}+\eta_{k'}\}}$$
$$< \frac{\sum_k \epsilon\exp\left[\left(\gamma(1-\tau)\frac{1}{R_k}P_k+\eta_k\right)\right]}{1+\sum_{k'}\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'}+\eta_{k'}\}} < \epsilon$$

where the equality follows from (11) and the last inequality is true for a small enough R_k because $\lim_{R_0\downarrow 0}\frac{\beta R}{1+\gamma(1-\tau)P_k\frac{1}{R_k}+\beta R_k} \to 0$. Therefore, $\lim_{R_k\downarrow 0} R\frac{\sum_{k'}\beta R'_{k'}\exp\{\alpha P_{k'}+\beta R_{k'}+\xi_{k'}\}}{1+\sum_{k'}\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'}+\eta_{k'}\}} = 0$.

Second, the second term in $f'(R_0)$ converges to infinity.

$$\frac{\sum_{k'}\left(\gamma(1-\tau)\frac{R'_{k'}}{R^2_{k'}}P_{k'}\right)\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'}+\eta_{k'}\}}{\left[1+\sum_{k'}\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'}+\eta_{k'}\}\right]^2}$$
$$= \frac{\sum_{k'}\left(\gamma(1-\tau)\frac{1}{R^2_{k'}}P_{k'}\frac{\exp\left[\left(\gamma(1-\tau)\frac{1}{R_k}P_k+\eta_k\right)-(\alpha P_k+\beta R_k+\xi_k)\right]}{1+\gamma(1-\tau)P_k\frac{1}{R_k}+\beta R_k}\right)\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'}+\eta_{k'}\}}{\left[1+\sum_{k'}\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'}+\eta_{k'}\}\right]^2}$$
$$= \sum_k \frac{\gamma(1-\tau)\frac{1}{R_k}P_k}{1+\gamma(1-\tau)P_k\frac{1}{R_k}+\beta R_k}\exp\left[-(\alpha P_k+\beta R_k+\xi_k)\right]\frac{\exp\{2(\gamma(1-\tau)\frac{1}{R_k}P_k+\eta_k)\}}{\left[1+\sum_{k'}\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'}+\eta_{k'}\}\right]^2}\frac{1}{R_k}$$
$$\to +\infty$$

because $\lim_{R_k\downarrow 0}\frac{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'}}{1+\gamma(1-\tau)P_k\frac{1}{R_k}+\beta R_k}\exp\{-(\alpha P_k+\beta R_k+\xi_k)\} = \exp\{-(\alpha P_k+\xi_k)\}$ and

$$\lim_{R_0\downarrow 0}\frac{\exp\{2(\gamma(1-\tau)\frac{1}{R_k}P_k+\eta_k)\}}{\left[1+\sum_{k'}\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'}+\eta_{k'}\}\right]^2} = [\frac{P_k\exp(\alpha P_k+\xi_k)}{\sum P_k\exp(\alpha P_k+\xi_k)}]^2 \tag{12}$$

In fact, consider

$$\lim_{R_0\downarrow 0}\frac{R_k}{R_{k'}} = \lim_{R_0\downarrow 0}\frac{R'_k}{R'_{k'}} = \lim_{R_0\downarrow 0}\frac{\frac{1}{R_{k'}}+\gamma(1-\tau)P_{k'}\frac{1}{R^2_{k'}}+\beta}{\frac{1}{R_k}+\gamma(1-\tau)P_k\frac{1}{R^2_k}+\beta} = \frac{P_{k'}}{P_k}[\lim_{R_0\downarrow 0}\frac{R_k}{R_{k'}}]^2$$

[4]Therefore, $f(0) = 0$. However, 0 is a pseudo solution that can be easily excluded because $s_0, \zeta_0 \in (0,1)$ thanks to the logit structure.

which implies that $\lim_{R_0\downarrow 0} \frac{R_k}{R_{k'}} = \frac{P_k}{P_{k'}}$ and

$$
\begin{aligned}
&\lim_{R_0\downarrow 0} \frac{\exp\{\gamma(1-\tau)\frac{1}{R_k}P_k + \eta_k\}}{\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'} + \eta_{k'}\}} \\
&= \lim_{R_0\downarrow 0} \exp\{(\gamma(1-\tau)\frac{1}{R_k}P_k + \eta_k) - (\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'} + \eta_{k'})\} \\
&= \lim_{R_0\downarrow 0} \exp\{(\log R_k - \log R_0 + [\alpha P_k + \beta R_k + \xi_k]) - (\log R_{k'} - \log R_0 + [\alpha P_{k'} + \beta R_{k'} + \xi_{k'}])\} \\
&= \lim_{R_0\downarrow 0} \exp\{\log \frac{R_k}{R_{k'}} + (\alpha P_k + \xi_k) - (\alpha P_{k'} + \xi_{k'})\} \\
&= \frac{P_k}{P_{k'}} \exp\{(\alpha P_k + \xi_k) - (\alpha P_{k'} + \xi_{k'})\}
\end{aligned}
$$

which implies (12).

We now show that "$f'(R_0)$ goes to zero" when R_k goes to infinity. Similarly, the firm term in $f'(R_0)$ converges to zero when R_0 goes to infinity because $\lim_{R_0\uparrow+\infty} \frac{\beta R}{1+\gamma(1-\tau)P_k\frac{1}{R_k}+\beta R_k} \to 0$. Moreover, the second term converges to zero because

$$
\begin{aligned}
\lim_{R_0\uparrow+\infty} \frac{\gamma(1-\tau)\frac{1}{R_k}P_k}{1+\gamma(1-\tau)P_k\frac{1}{R_k}+\beta R_k} &= 0 \\
\lim_{R_0\uparrow+\infty} \exp[-(\alpha P_k + \beta R_k + \xi_k)] &= 0 \\
\lim_{R_0\uparrow+\infty} \frac{\exp\{2(\gamma(1-\tau)\frac{1}{R_k}P_k + \eta_k)\}}{\left[1+\sum_{k'}\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'}+\eta_{k'}\}\right]^2} &= \frac{\exp\{2\eta_k\}}{[1+\sum_{k'}\exp\{\eta_{k'}\}]^2} \\
\lim_{R_0\uparrow+\infty} \frac{1}{R_k} &= 0
\end{aligned}
$$

□

Lemmas 1 and 2 suggest there exists at least an interior equilibrium where R_k are finite.

Theorem 2. *If $\beta R < 1$, a unique interior equilibrium exists, which can be found by contraction mapping f.*

Proof. We show that $f'(R_0) \in (0,1)$ if $R_0 = f(R_0)$; the slope of f is smaller than the

45-degree line at any solution. The result then follows Lemma 3.

$$
\begin{aligned}
&f'(R_0)\\
&= R\frac{\sum_{k'}\beta R'_{k'}\exp\{\alpha P_{k'}+\beta R_{k'}+\xi_{k'}\}}{1+\sum_{k'}\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'}+\eta_{k'}\}}\\
&\quad + R\frac{1+\sum_{k'}\exp\{\alpha P_{k'}+\beta R_{k'}+\xi_{k'}\}}{\left[1+\sum_{k'}\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'}+\eta_{k'}\}\right]^2}\left[\sum_{k'}\left(\gamma(1-\tau)\frac{R'_{k'}}{R_{k'}^2}P_{k'}\right)\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'}+\eta_{k'}\}\right]\\
&= \left[\sum_{k'}\left(\frac{\beta R}{1+\gamma(1-\tau)P_k\frac{1}{R_k}+\beta R_k}+\gamma(1-\tau)\frac{R'_{k'}}{R_{k'}^2}P_{k'}R_0\right)\frac{\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'}+\eta_{k'}\}}{1+\sum_{k'}\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'}+\eta_{k'}\}}\right]\\
&= \sum_{k'}\left(\frac{\beta R}{1+\gamma(1-\tau)P_k\frac{1}{R_k}+\beta R_k}+(\frac{1}{R_0}-\beta R'_k-\frac{R'_k}{R_k})R_0\right)\frac{\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'}+\eta_{k'}\}}{1+\sum_{k'}\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'}+\eta_{k'}\}}\\
&= \sum_{k'}\left(\frac{\beta R}{1+\gamma(1-\tau)P_k\frac{1}{R_k}+\beta R_k}-\frac{(\beta+\frac{1}{R_k})\frac{1}{R_0}}{\frac{1}{R_k}+\gamma(1-\tau)P_k\frac{1}{R_k^2}+\beta}R_0+1\right)\frac{\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'}+\eta_{k'}\}}{1+\sum_{k'}\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'}+\eta_{k'}\}}\\
&= \sum_{k}\left(\frac{\beta R+\gamma(1-\tau)P_k\frac{1}{R_k}}{1+\gamma(1-\tau)P_k\frac{1}{R_k}+\beta R_k}\right)\frac{\exp\{\gamma(1-\tau)\frac{1}{R_k}P_k+\eta_k\}}{1+\sum_{k'}\exp\{\gamma(1-\tau)\frac{1}{R_{k'}}P_{k'}+\eta_{k'}\}}
\end{aligned}
$$

where the second equation follows from (11) and $R_0 = f(R_0)$, the third follows from (10), the fourth equation follows from (11) and the rest are trivial.

If $\beta R < 1 + \beta R_k$ for all k, we have $f'(R_0) < 1$. So a sufficient condition is $\beta R < 1$. □

Lemma 3. *If $f'(R_0) < 1$ for any $R_0 > 0$ such that $R_0 = f(R_0)$, f has one unique interior solution.*

Proof. Define $x_L = \inf\{x > 0 : f(x) = x\}$, which must be strictly positive. Because $f'(x_L) < 1$, there exists $\epsilon > 0$ such that $f(x) < x$ when $x \in (x_L, x_L + \epsilon)$. Denote $x_R = \sup\{f(x) < x\}$. We want to show that $x_R = +\infty$. If x_R is finite, we have $f(x_R) = x_R$ and $f'(x_R) < 1$. Therefore, there exists a small interval $(x_R - \epsilon', x_R)$ such that $f(x) > x$, which contradicts the definition of x_R. □

5 Results

There are three advantages of having a PoP. Firstly, a PoP possesses its own customer base, generating additional synergy for all affiliated operating platforms. In our model, this is represented as the term ξ_G in equation 3. Secondly, a PoP consolidates all customers and drivers together, allowing an operating platform that joins the PoP to be matched with a larger pool of potential customers and benefit from a stronger cross-side network effect.

Thirdly, a PoP provides operating platforms with an opportunity to enter the market at a reduced cost. In our model, this is reflected as a lower entry cost κ. In the baseline model, we assume that the entrants are symmetrical, meaning they have the same values for ξ_E and η_E. We assume the presence of a dominant incumbent with higher ξ_I and η_I, mirroring the reality of the incumbent Didi Global's dominance in the market, while smaller ride-hailing platforms attempt to enter the market.

To present comparative statistics regarding our model, we analyze the impact of a PoP on the pricing and entry decisions of operating platforms. We investigate how the equilibrium outcomes are influenced by variations in the synergy ξ_G created by the PoP. A higher value of ξ_G indicates a larger customer base for the PoP or a greater level of synergy generated for users. Specifically, we explore two scenarios. In the first scenario, the PoP establishes a fixed commission rate while the value of ξ_G changes. In the second scenario, the PoP dynamically adjusts the commission rate in response to changes in the value of ξ_G.

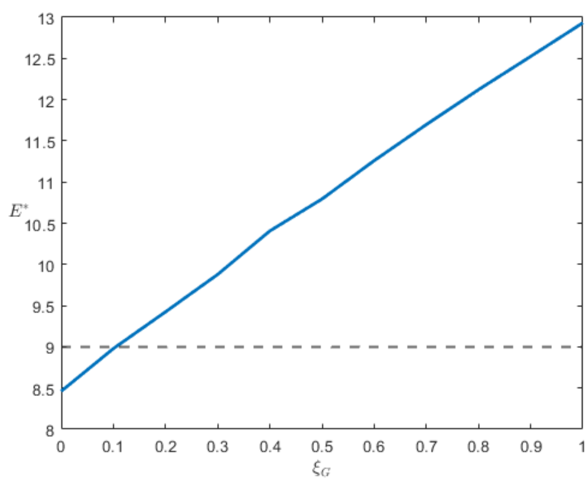
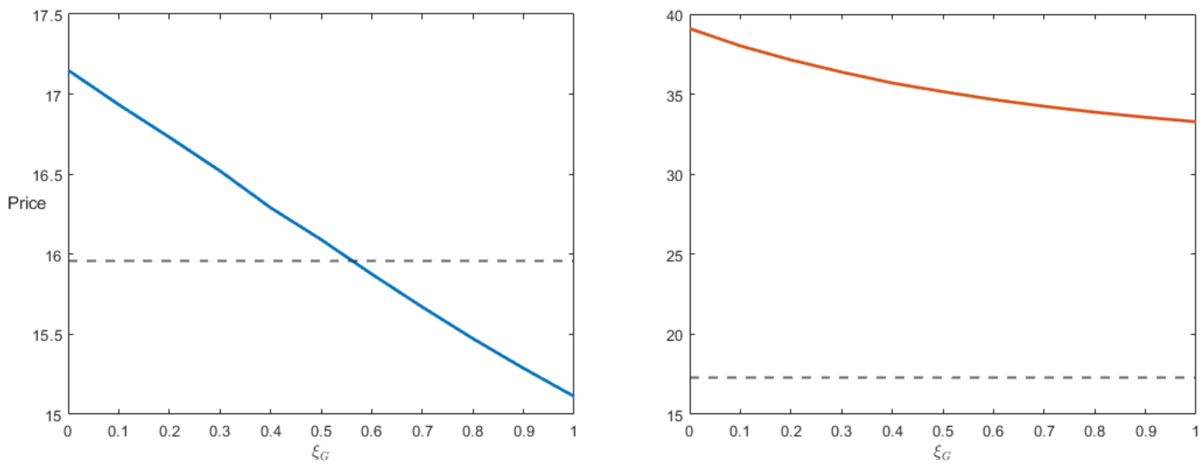


Figure 4: Number of Entrants with Fixed Commission Rate of the PoP

Firstly, we demonstrate the impact of a PoP on the equilibrium number of entrants, comparing cases with and without a PoP. Figure 4 illustrates the number of entrants when a fixed commission rate is set by the PoP. The dashed line represents the equilibrium number of entrants in the absence of a PoP. It is evident that when the PoP generates greater synergy, represented by a higher value of ξ_G, the equilibrium number of entrants increases. Conversely, if the synergy is minimal, the presence of a PoP leads to a lower number of entrants in equilibrium compared to the scenario without a PoP. This is because while a PoP reduces the entry cost and generates consumer synergy, it also softens competition among operating platforms by imposing a commission fee. Consequently, operating platforms increase their ride fares as a result of the presence of the PoP. To further illustrate this point, next we analyze the equilibrium prices set by both the incumbent and the entrants.



(a) Price of the Incumbent

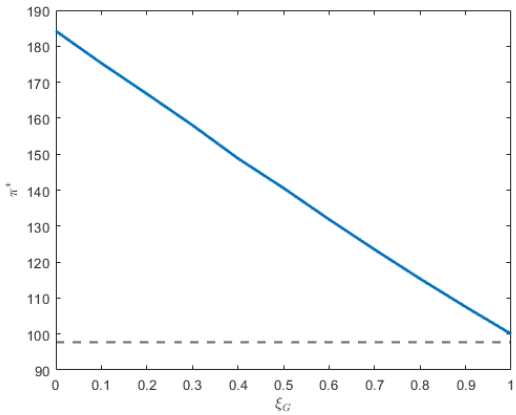
(b) Price of the Entrants

Figure 5: Equilibrium Prices with Fixed Commission Rate

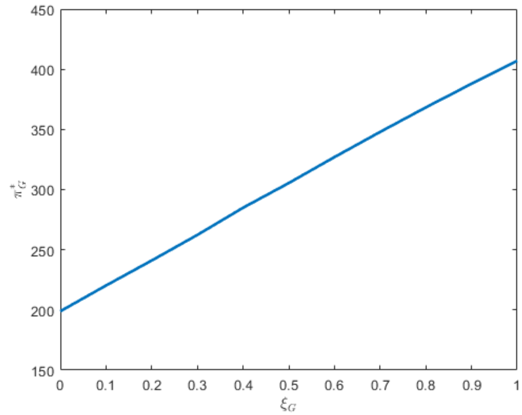
Panel (a) in Figure 5 illustrates the price set by the incumbent in the presence of the PoP. The dashed line represents the price of the incumbent in the absence of a PoP but with free entry of other ride-hailing platforms. We can see that initially, in the presence of a PoP, the incumbent sets a higher price compared to the scenario without a PoP. However, as the synergy generated by the PoP increases, the incumbent lowers its price due to greater competition from a larger number of entrants. Nevertheless, Panel (b) in Figure 5 highlights that the prices set by entrants are significantly higher when a PoP is present compared to the scenario without a PoP. This occurs because the PoP softens competition among entrants by imposing a commission rate on operating platforms. Consequently, the ultimate ride fares faced by consumers are considerably higher in the presence of a PoP.

Figure 6 illustrates the profits of both the incumbent and the PoP as the value of ξ_G varies. In Panel (a), it is evident that the incumbent experiences higher profits in the presence of a PoP. This can be attributed to the fact that the PoP mitigates competition among different operating platforms. Moreover, as expected, when the value of ξ_G increases, the PoP earns higher profits.

Table 3 examines the welfare effects of having a PoP. The numbers in the table represent the percentage change between the equilibrium outcome with a PoP and the outcome without a PoP. The results indicate a significant decrease in consumer surplus when a PoP is present. Despite the PoP generating synergy and larger network effects for consumers, it also reduces competition, resulting in increased ride fares. Nevertheless, on the other hand,



(a) Profit of the Incumbent



(b) Profit of the PoP

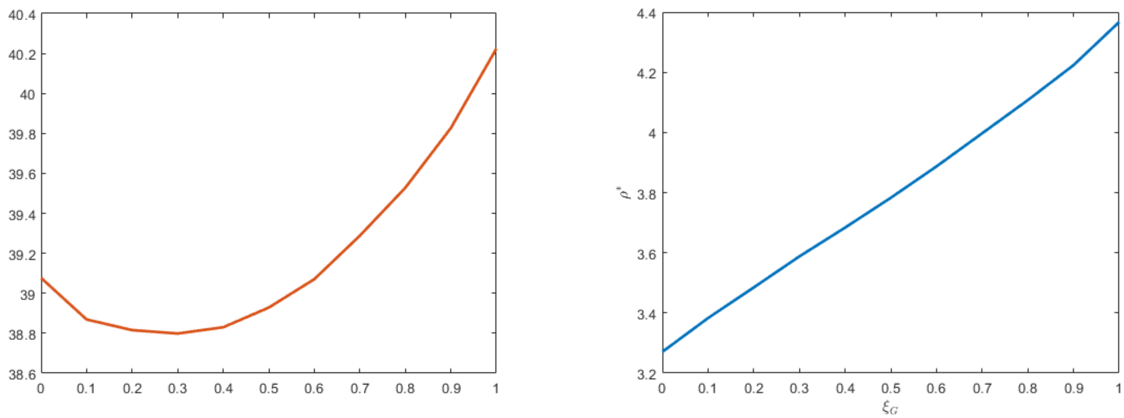
Figure 6: Equilibrium Profits with Fixed Commission Rate

when a substantial synergy is generated by the PoP, the number of entrants increases by approximately 44%. Drivers benefit from both higher ride fares and an increased number of operating platforms, resulting in a welfare increase of approximately 7%. Additionally, the incumbent's profit is greater when a PoP is implemented compared to the scenario without one. The results presented in Table 3 reveal an intriguing phenomenon. Despite the presence of a PoP leading to a higher number of operating platforms, consumers actually experience the adverse effects of "more" competition.

Table 3: Changes in Welfare with Fixed Commission Rate

ξ_G	Consumer Surplus	Driver Surplus	Profit of the Incumbent	Number of Entrant
0	-46.9%	4.4%	88.3%	-6.0%
0.1	-45.5%	4.9%	79.2%	-0.3%
0.2	-44.0%	5.3%	70.4%	4.7%
0.3	-42.4%	5.7%	61.5%	9.7%
0.4	-40.7%	6.2%	52.1%	15.6%
0.5	-38.9%	6.5%	43.7%	19.9%
0.6	-36.9%	6.8%	34.8%	25.1%
0.7	-34.9%	7.1%	26.2%	29.9%
0.8	-32.8%	7.3%	17.9%	34.6%
0.9	-30.5%	7.5%	9.9%	39.1%
1	-28.2%	7.7%	2.2%	43.6%

Next, we investigate the impact of allowing the PoP to optimize the commission rate with varying values of the created synergy. In this scenario, the commission fee charged, represented as ρ^*, varies in response to changes in the value of ξ_G.



(a) Price of the Entrant

(b) Commission Rate of the PoP

Figure 7: Equilibrium Profits with Optimized Commission Rate

Figure 7 illustrates the prices set by the entrants as well as the optimal commission fee charged by the PoP. When comparing Panel (a) in Figure 7 with Panel (b) in Figure 5, a significant difference can be observed. Specifically, the price set by the entrant no longer exhibits a strictly decreasing trend as the value of ξ_G increases. This can be attributed to the PoP opting for a higher commission rate, as depicted in Panel (b) of Figure 7. Consequently, when the synergy is greater, the PoP chooses to charge higher ride fares as it becomes more appealing in the market.

Next, we compare the equilibrium outcomes between the scenario with an optimized commission rate and the scenario with a fixed commission rate. Figure 8 depicts the number of entrants in equilibrium in both scenarios. It can be observed that, qualitatively speaking, the number of entrants increases as the synergy of the PoP grows larger. However, there is a significant reduction in the number of entrants when the PoP optimizes the commission rate. This is attributed to the fact that when ξ_G is larger, the PoP charges a higher commission rate, which in turn leads to lower profits for the operating platforms. Consequently, this offsets the benefits generated by the PoP's synergy and results in a smaller number of entrants.

Figure 9 compares the equilibrium prices in the two scenarios. It can be observed that when the PoP optimizes the commission rate, it leads to higher prices for both the entrant and the incumbent. In Panel (b) of Figure 9, it is evident that even when the synergy created by the PoP is larger, the price set by the incumbent remains higher compared to the scenario without a PoP. This demonstrates that the PoP not only mitigates competition

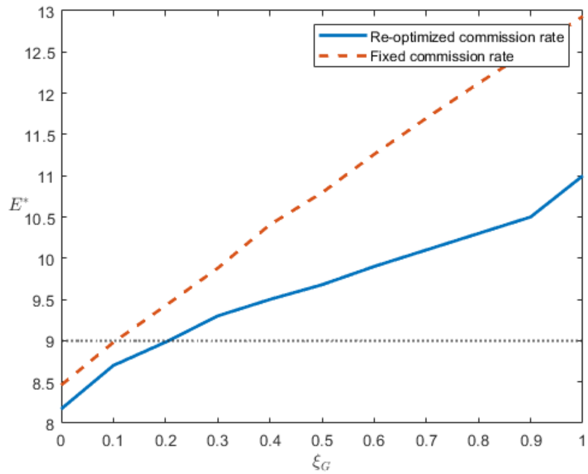
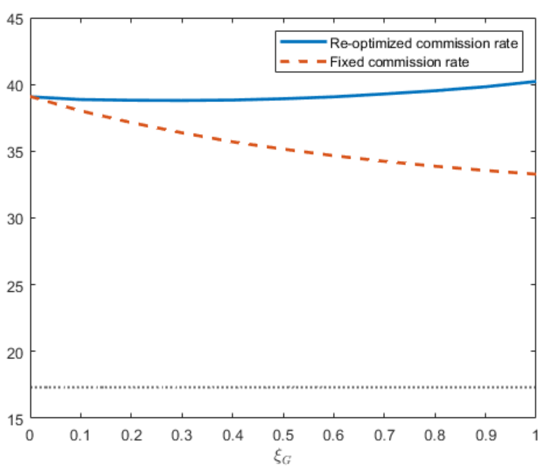
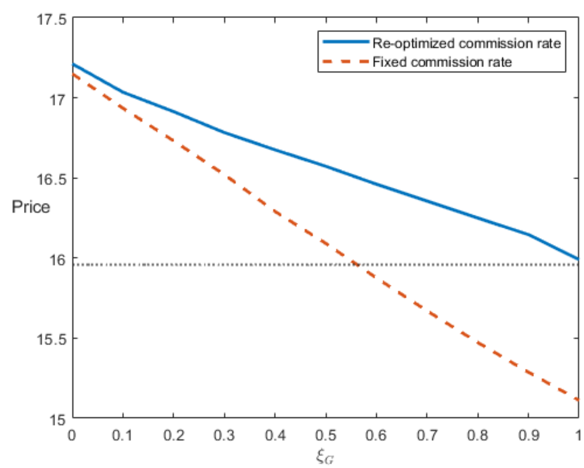


Figure 8: Number of Entrants

among operating platforms but also exacerbates the situation when allowed to optimize the commission rate, resulting in even higher ride fares for consumers.



(a) Price of the Entrant



(b) Price of the Incumbent

Figure 9: Equilibrium Prices

Due to the higher prices resulting from allowing the PoP to optimize the commission rate, the incumbent experiences increased profits compared to the scenario where the PoP charges a fixed commission rate with varying values of ξ_G. Nonetheless, in both cases, the incumbent achieves higher profits compared to the scenario without a PoP, as shown in Figure 10.

Lastly, we present the welfare effects in Table 4. The results are qualitatively similar to those obtained when the PoP charges a fixed commission rate, as shown in Table 3. However, when the PoP optimizes the commission rate, the ride fares further increase. Consequently,

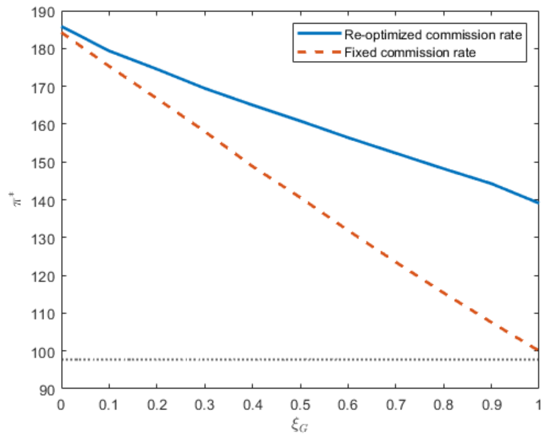


Figure 10: Profit of the Incumbent

the consumer welfare experiences an even greater decrease compared to the results in Table 3, owing to softened competition and the higher commission rate imposed by the PoP. On the other hand, both drivers and the incumbent benefit from these further increased prices, resulting in greater surplus gains. Regarding the level of competition in equilibrium, we observe that the total number of entrants is smaller when the PoP is allowed to optimize the commission rate compared to the scenario where the PoP only charges a fixed rate.

Table 4: Changes in Welfare with Optimized Commission Rate

ξ_G	Consumer Surplus	Driver Surplus	Profit of the Incumbent	Number of Entrants
0	-46.8%	4.1%	89.9%	-9.2%
0.1	-46.1%	4.7%	83.3%	-3.4%
0.2	-45.3%	5.1%	78.4%	-0.2%
0.3	-44.5%	5.5%	73.2%	3.3%
0.4	-43.7%	5.8%	68.7%	5.5%
0.5	-42.9%	6.2%	64.4%	7.5%
0.6	-42.1%	6.5%	59.9%	10.0%
0.7	-41.3%	6.9%	55.7%	12.2%
0.8	-40.4%	7.2%	51.5%	14.4%
0.9	-39.6%	7.6%	47.4%	16.6%
1	-38.9%	8.2%	42.1%	22.2%

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